

# Photon-assisted entanglement creation by minimum-error generalized quantum measurements in the strong coupling regime

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In generalization of the hybrid quantum repeater model of van Loock et al. [1] we explore possibilities of entangling two distant material qubits with the help of a single-mode optical radiation field in the strong quantum electrodynamical coupling regime of almost resonant interaction. The optimum generalized field measurements are determined which are capable of preparing a two-qubit Bell state by postselection with minimum error. It is demonstrated that in the strong coupling regime some of the recently found limitations of the non-resonant weak coupling regime can be circumvented successfully due to characteristic quantum electrodynamical quantum interference effects. In particular, in the absence of photon loss it is possible to postselect two-qubit Bell states with fidelities close to unity by a proper choice of the relevant interaction time. Even in the presence of photon loss this strong coupling regime offers interesting perspectives for creating spatially well separated Bell pairs with high fidelities and with high repetition rates which are relevant for future realizations of quantum repeaters.

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## I. INTRODUCTION

Entanglement is a primary resource for quantum technology [2]. For applications in quantum communication, such as quantum key distribution, for example, the generation of well-controlled entanglement between spatially separated quantum systems is of crucial importance. For this purpose quantum repeaters [3] are needed which counteract the destructive influence of uncontrolled environmental interactions.

Since the early work of Briegel et al. [4, 5] there have been numerous theoretical proposals suggesting different physical platforms for realizing quantum repeaters [3]. They are based on the main idea of creating entanglement between quantum systems over large distances with the help of a chain of many uncorrelated pairs of quantum systems each of which is entangled over a significantly shorter distance only. Performing appropriate Bell measurements on each of the two qubits of adjacent entangled pairs it is possible to swap the already existing short-distance entanglement to the largely separated outermost quantum systems of such a chain.

From the experimental point of view the realization of a quantum repeater still constitutes a major technological challenge. Essential for any such realization are two prerequisites, namely efficient physical mechanisms for generating highly entangled pairs of quantum systems with high success probabilities and high repetition rates and optimal ways for implementing complete Bell measurements accurately. It has been demonstrated theoretically [3] that the exchange of photons provides powerful means for entangling material quantum systems at least over distances of moderate lengths, say a few kilometers, thus suggesting practicable solutions for the first prerequisite.

An interesting example in this respect is the recent theoretical proposal of van Loock et al. [1, 6, 7] of a

hybrid quantum repeater based on continuous variables. It suggests the exchange of a single-mode coherent state of an optical radiation field for purposes of entangling spatially separated material qubits. It takes advantage of the fact that experimentally these field states can be controlled well and that they can also be produced with high repetition rates. The main idea of this proposal is to entangle a single-mode optical radiation field with two initially uncorrelated material qubit systems quantum electromagnetically and to create entanglement between these two qubits by an appropriate measurement of the quantum state of the radiation field which post-selects a Bell state of the two material qubits. In their original proposal van Loock et al. [1, 6] discuss cases in which the qubit systems couple to the radiation field in a non-resonant way so that their quantum electrodynamical interaction is weak and can be described perturbatively. Although offering a transparent theoretical description this perturbative regime of the electromagnetic coupling also causes major limitations as far as the achievable degree of entanglement is concerned. These limitations can be traced back to the fact that the relevant field states which have to be distinguished in order to postselect a Bell state of the material qubit pair are not orthogonal. Thus, these field states cannot be distinguished perfectly by any quantum measurement so that the entanglement of the postselected material two-qubit state is never perfect. In view of these developments the natural question arises whether it is possible to circumvent these limitations and to provide a physical mechanism which is capable of producing entangled two-qubit pairs not only with a high repetition rate and high success probability but also with arbitrarily high degree of entanglement. For any future realization of a quantum repeater such a mechanism for creating at least short- to intermediate-distance entanglement between two qubits is useful as it is expected to increase final rates of pro-

ducing long-distance entanglement by subsequent entanglement swapping and quantum state purification significantly. It is a main aim of this paper to address this question.

In the following it is demonstrated that the strong coupling limit of the quantum electrodynamical interaction offers significant advantages for photon-assisted entanglement creation between material quantum systems. Coupling two material few-level systems almost resonantly to a single mode of the quantized radiation field the performance of entanglement creation processes, such as the one originally proposed by van Loock et al. [1], can be improved significantly. This way it is possible to circumvent previously discussed limitations which result from the restriction of the quantum electrodynamical interaction to the weak coupling limit. In contrast, in the strong coupling limit it is even possible to realize physical situations in which two material quantum systems can be postselected in a perfect Bell state by an appropriate von Neumann measurement of the quantized radiation field. However, for this purpose it is necessary to control the relevant interaction times between the quantized radiation field and the two material few-level systems appropriately. For sufficiently intense radiation fields these interaction times can even be chosen so short that effects of spontaneous emission of photons into other modes of the radiation field can be neglected so that a major decoherence mechanism can be eliminated and all advantages of quantum interferences can be exploited.

This paper is organized as follows. In Sec.II we introduce the quantum electrodynamical model in which two elementary three-level systems interact with a single mode of the optical radiation field. Furthermore, we develop the general framework for describing the post-selection procedure which prepares distant qubits in a Bell state by an optimal generalized field measurement which introduces minimum errors. Numerical results are presented for characteristic quantities which quantify the success with which a Bell pair is prepared, its fidelity and the minimum error with which this postselection can be achieved. Whereas Sec.II discusses cases in which the propagation of the optical radiation field between the two qubits is ideal, in Sec.III modifications originating from photon loss during this propagation are taken into account.

## II. PHOTON-ASSISTED ENTANGLEMENT CREATION

In this section we discuss a quantum electrodynamical model describing a typical interaction scenario in which two spatially separated elementary (material) three-level systems are entangled by their almost resonant interaction with a single-mode of the quantized radiation field. In the weak coupling limit this model has been introduced by van Loock et al.[1] as an efficient method for generating distantly entangled qubit pairs. Our main aim in

this chapter is to explore the dynamics of this system in the strong coupling limit in which both three-level systems interact with the quantized radiation field almost resonantly and to investigate to which extent minimum-error field measurements are capable of postselecting a high-fidelity Bell state efficiently.

### A. The quantum electrodynamical model

In analogy to the original quantum repeater proposal of van Loock et al.[1] we consider two spatially separated elementary (material) three-level systems  $A$  and  $B$ , e.g. trapped atoms or ions, with energy eigenstates  $|0\rangle_i, |1\rangle_i$ , and  $|2\rangle_i$  ( $i \in \{A, B\}$ ) and associated energies  $E_0, E_1$ , and  $E_2$  (compare with Fig.1). The energy eigenstates  $|1\rangle_i$  and  $|2\rangle_i$  with  $i \in \{A, B\}$  are assumed to be of opposite parity and to be coupled almost resonantly to a radiation field of frequency  $\omega$  by an allowed optical dipole transition. Each of these two material quantum systems is assumed to be surrounded by a cavity so that each of these systems interacts with a single cavity mode only. Furthermore, these cavities are assumed to be linked by

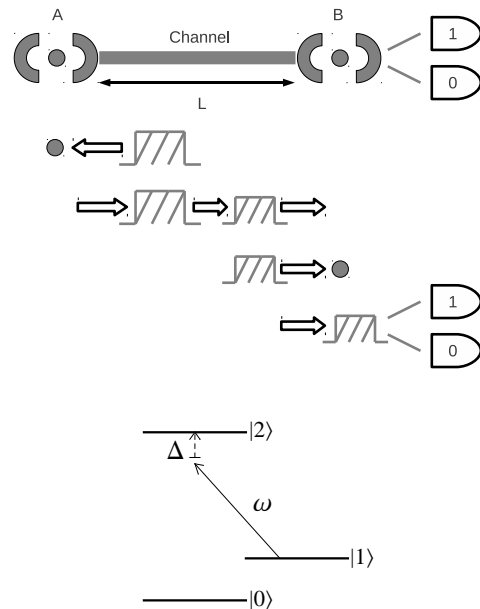


FIG. 1: Schematic representation of photon-assisted entanglement creation: After the first interaction of duration  $\tau$  of a photon wave packet with the material quantum system  $A$  inside a cavity this wave packet propagates a distance  $L = cT$  with speed  $c$  through an optical fiber and interacts in a similar way with the second material quantum system  $B$ . Immediately afterwards this photon wave packet is projected by a minimum-error two-valued POVM measurement with measurement results  $m \in \{1, 0\}$ . The measurement result  $m = 1$  prepares both material quantum systems in a Bell state  $|\Psi^+\rangle$  with success probability  $P_{Bell}$  and with fidelity  $F_{opt}$ .

an optical fiber through which the photons of one cavity can travel to the other cavity. The coupling of the far-detuned third level  $|0\rangle$  to the radiation field is assumed to be negligible. The energy eigenstates  $|0\rangle$  and  $|1\rangle$  can be two energetically low lying atomic states of different parity, for example, whose energy separation is determined by hyperfine splitting. These two states serve as a qubit. It is the main purpose of the subsequent discussion to entangle the two qubits of systems  $A$  and  $B$  formed by the states  $|0\rangle_i$  and  $|1\rangle_i$  ( $i \in \{A, B\}$ ) by the strong quantum electrodynamical coupling of the optical radiation field to the states  $|1\rangle_i$  and  $|2\rangle_i$  ( $i \in \{A, B\}$ ) of these two three-level systems. For this purpose we assume, in analogy to the originally proposed hybrid quantum repeater model of van Loock et al. [1], that the interaction between these two three-level systems and the optical radiation field can be described in the framework of an effective single-mode description with the help of a slowly varying amplitude approximation.

In order to entangle the two material quantum systems  $A$  and  $B$  with the help of the optical radiation field we consider a Ramsey-type interaction scenario as depicted schematically in Fig.1. In a first step the single-mode radiation field, which is prepared in a coherent state  $|\alpha\rangle$  initially, interacts with the three-level system  $A$  during a time interval of duration  $\tau$ . Subsequently, the entangled matter-field system evolves freely without any coupling for a time interval of duration  $T$ . This part of the dynamics describes the propagation of the optical radiation field from system  $A$  to the distant system  $B$ . In the third and final step of the interaction this single-mode optical radiation field interacts with system  $B$  during a time interval of duration  $\tau$ . If we neglect spontaneous emission of photons of the three-level systems into other modes of the radiation field the dynamics of this three-stage process can be described by the Hamiltonians

$$\begin{aligned} 0 \leq t \leq \tau: \quad \hat{H} &= \hat{H}^{(A)}, \\ \tau < t < T + \tau: \quad \hat{H} &= \hat{H}_A + \hat{H}_B + \hbar\omega\hat{a}^\dagger\hat{a}, \\ T + \tau \leq t \leq T + 2\tau: \quad \hat{H} &= \hat{H}^{(B)} \end{aligned} \quad (1)$$

with  $\hat{H}^{(j)} = \hat{H}_A + \hat{H}_B + \hbar\omega\hat{a}^\dagger\hat{a} + \hbar g\hat{a}|2\rangle_{jj}\langle 1| + \hbar g^*\hat{a}^\dagger|1\rangle_{jj}\langle 2|$

and  $j \in \{A, B\}$ . Thereby, the unperturbed Hamiltonians of systems  $A$  and  $B$  are given by  $\hat{H}_i = \hbar\omega_0|0\rangle_{ii}\langle 0| + \hbar(\omega_{21}/2)|2\rangle_{ii}\langle 2| - \hbar(\omega_{21}/2)|1\rangle_{ii}\langle 1|$  with  $i \in \{A, B\}$  and with  $\omega_{21} = (E_2 - E_1)/\hbar$  and  $\hbar\omega_0 = E_0 + (E_1 + E_2)/2$ . The unperturbed Hamiltonian governing the dynamics of the single-mode radiation field is given by  $\hat{H}_F = \hbar\omega\hat{a}^\dagger\hat{a}$  with the creation and destruction operators  $\hat{a}^\dagger$  and  $\hat{a}$  of the single-mode radiation field. In Eq.(1) the interaction between this optical field mode and the material systems  $A$  and  $B$  is described in the dipole and rotating wave approximation. It is characterized by the coupling parameters  $g_i = -{}_i\langle 2|\hat{\mathbf{d}}|1\rangle_i\sqrt{\hbar\omega/(2\epsilon_0)}\mathbf{g}_i(\mathbf{x}_i)/\hbar$  which involve the transition dipole moments  $\langle 2|\hat{\mathbf{d}}|1\rangle_i$  and the normalized mode functions of the single mode radiation field  $\mathbf{g}_i(\mathbf{x})$  in their respective cavities. For simplicity in the following we concentrate on cases of symmetric couplings in which  $g_A = g_B = g$ . The modulus of this characteristic coupling strength defines the resonant vacuum Rabi frequency, i.e.  $\Omega_{vac} = |g|$  [8]. In Eq.(1) it is assumed that the dynamical evolution of the optical radiation field during its propagation through the optical fiber from system  $A$  to system  $B$  is ideal so that it is not affected by any uncontrolled couplings to an environment. Realistic effects of photon loss during this propagation will be discussed in Sec.III.

Initially, the quantum state of the two systems  $A$  and  $B$  and the single-mode radiation field is assumed to be separable and of the form

$$|\Psi(t=0)\rangle = \frac{|0\rangle_A + |1\rangle_A}{\sqrt{2}} \otimes \frac{|0\rangle_B + |1\rangle_B}{\sqrt{2}} \otimes |\alpha\rangle. \quad (2)$$

Thus, this state can be prepared by local operations acting on the initially uncorrelated material three-level systems. The single-mode radiation field is assumed to be prepared in the coherent state  $|\alpha\rangle$  with the mean photon number  $\bar{n} = |\alpha|^2$ .

It is straightforward to determine the quantum state  $|\Psi(t)\rangle$  at time  $t = T + 2\tau$  after the Ramsey-type interaction sequence described above. It is given by

$$\begin{aligned} |\Psi(t)\rangle = & \frac{1}{2}|0\rangle_A|0\rangle_B|\alpha e^{-i\omega t}\rangle e^{-i\Phi_{00}} + |g_1(t)\rangle|\Psi^+\rangle_{AB}e^{-i\Phi_{10}} + |g_2(t)\rangle(|0\rangle_A|2\rangle_B e^{-i\Phi_{02}} + |2\rangle_A|0\rangle_B e^{-i\Phi_{20}}) + \\ & |g_3(t)\rangle|1\rangle_A|1\rangle_B e^{-i\Phi_{11}} + |g_4(t)\rangle|2\rangle_A|1\rangle_B e^{-i\Phi_{21}} + |g_5(t)\rangle|1\rangle_A|2\rangle_B e^{-i\Phi_{12}} + |g_6(t)\rangle|2\rangle_A|2\rangle_B e^{-i\Phi_{22}} \end{aligned} \quad (3)$$

with the Bell state  $|\Psi^+\rangle_{AB} = (|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)/\sqrt{2}$  involving the two distant qubits of systems  $A$  and  $B$ . The time-dependent phases  $\Phi_{00} = 2\omega_0 t$ ,  $\Phi_{10} = \omega_0 t - \omega\tau/2 - \omega_{21}(T + \tau)/2$ ,  $\Phi_{20} = \omega_0 t + \omega\tau/2 + \omega_{21}(T + \tau)/2$ ,  $\Phi_{02} = \omega_0 t + \omega\tau/2 - \omega_{21}(T + \tau)/2 + \omega(T + \tau)$ ,  $\Phi_{11} = -\omega_{21}(T +$

$\tau) - \omega\tau$ ,  $\Phi_{12} = -\omega_{21}(T + \tau) + \omega(T + \tau)$ ,  $\Phi_{21} = 0$ ,  $\Phi_{22} = \omega t$  characterise the interferences appearing in this Ramsey-type interaction scenario. The (unnormalized) pure field states  $|g_j(t)\rangle$  ( $j = 1, \dots, 6$ ) entering Eq.(3) are defined by

$$\begin{aligned}
|g_1(t)\rangle &= \sum_{n=0}^{\infty} e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}} \frac{1}{\sqrt{2}} \left( \cos(\Omega(n)\tau) + i \frac{\Delta}{2\Omega(n)} \sin(\Omega(n)\tau) \right) e^{-i\omega n t} |n\rangle, \\
|g_2(t)\rangle &= - \sum_{n=0}^{\infty} e^{-|\alpha|^2/2} \frac{\alpha^{(n+1)}}{\sqrt{(n+1)!}} \frac{ig\sqrt{n+1}}{\Omega(n+1)} \frac{1}{2} \sin(\Omega(n+1)\tau) e^{-i\omega n t} |n\rangle, \\
|g_3(t)\rangle &= \sum_{n=0}^{\infty} e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}} \frac{1}{2} \left( \cos(\Omega(n)\tau) + i \frac{\Delta}{2\Omega(n)} \sin(\Omega(n)\tau) \right)^2 e^{-i\omega n t} |n\rangle, \\
|g_4(t)\rangle &= - \sum_{n=0}^{\infty} e^{-|\alpha|^2/2} \frac{\alpha^{(n+1)}}{\sqrt{(n+1)!}} \frac{ig\sqrt{n+1}}{\Omega(n+1)} \sin(\Omega(n+1)\tau) \frac{1}{2} \left( \cos(\Omega(n)\tau) + i \frac{\Delta}{2\Omega(n)} \sin(\Omega(n)\tau) \right) e^{-i\omega n t} |n\rangle, \\
|g_5(t)\rangle &= - \sum_{n=0}^{\infty} e^{-|\alpha|^2/2} \frac{\alpha^{(n+1)}}{\sqrt{(n+1)!}} \frac{ig\sqrt{n+1}}{\Omega(n+1)} \frac{\sin(\Omega(n+1)\tau)}{2} \left( \cos(\Omega(n+1)\tau) + i \frac{\Delta}{2\Omega(n+1)} \sin(\Omega(n+1)\tau) \right) e^{-i\omega n t} |n\rangle, \\
|g_6(t)\rangle &= - \sum_{n=0}^{\infty} e^{-|\alpha|^2/2} \frac{\alpha^{(n+2)}}{\sqrt{(n+2)!}} \frac{g\sqrt{n+1}}{\Omega(n+1)} \sin(\Omega(n+1)\tau) \frac{g\sqrt{n+2}}{\Omega(n+2)} \frac{1}{2} \sin(\Omega(n+2)\tau) e^{-i\omega n t} |n\rangle
\end{aligned} \tag{4}$$

with the normalized photon number states  $|n\rangle$  ( $n \in \mathbb{N}_0$ ). The parameter  $\Omega(n) := \sqrt{\Delta^2/4 + |g|^2 n}$  denotes the effective Rabi frequency associated with the photon number  $n$  of the optical radiation field and  $\Delta := \omega_{21} - \omega$  is the detuning from resonance.

The quantum state of Eq.(3) yields a complete description of the interaction between the material quantum systems  $A$  and  $B$  and the single-mode optical radiation field in the idealized case of free evolution during the propagation of the optical radiation field through the fiber from system  $A$  to system  $B$ . In particular, it clearly exhibits the resulting entanglement between the material systems  $A$  and  $B$  on the one hand and the radiation field on the other hand. In the weak coupling limit of large detunings from resonance this latter entanglement has been used in the proposal by van Loock et al. [1] for creating an entangled Bell state  $|\Psi^+\rangle_{AB}$  by projecting out the field state  $|g_1(t)\rangle$  by a generalized positive-operator-valued quantum measurement (POVM) [9–11] performed on the optical radiation field. However, in the weak coupling limit the field states  $|g_i(t)\rangle$  ( $i = 1, \dots, 6$ ) are not orthogonal so that the field state  $|g_1(t)\rangle$  cannot be distinguished perfectly from the other field states. This limits the achievable entanglement of the entangled state of the two material qubits significantly. In the subsequent subsection it will be demonstrated that in the strong coupling limit of almost resonant quantum electrodynamical coupling these limitations can be circumvented and in certain dynamical regimes even perfect Bell states  $|\Psi^+\rangle_{AB}$  can be prepared by suitable quantum measurements on the optical field.

## B. Postselection of Bell states by minimum-error field measurements

In this section we investigate to which extent the Ramsey-type interaction scenario discussed in the previous subsection can be optimized in order to prepare a maximally entangled Bell state between the distant material quantum systems  $A$  and  $B$  by an appropriate minimum-error POVM measurement of the optical field. This aspect is of particular interest for future realizations of quantum repeaters which require high-fidelity Bell pairs as a resource. It is demonstrated that in the ideal case in which the propagation of the optical radiation field through the fiber from system  $A$  to system  $B$  is not affected by uncontrolled environmental influences in the strong coupling limit of almost resonant interaction high-fidelity Bell states of systems  $A$  and  $B$  can be prepared provided the relevant interaction times are controlled appropriately.

Let us start from the pure quantum state  $|\Psi(t)\rangle$  of Eq.(3) which describes the entanglement of the matter-field system after the Ramsey-type interaction scenario. The resulting field state  $\hat{\rho}_F(t)$  is obtained by tracing out the material degrees of freedom, i.e.

$$\hat{\rho}_F(t) = \text{Tr}_{AB}\{|\Psi(t)\rangle\langle\Psi(t)|\} = p\hat{\rho}_1 + (1-p)\hat{\rho}_2 \tag{5}$$

with the field states

$$\begin{aligned}
\hat{\rho}_1 &= \frac{|g_1(t)\rangle\langle g_1(t)|}{p}, \\
\hat{\rho}_2 &= \frac{\frac{1}{4}|\alpha e^{-i\omega t}\rangle\langle\alpha e^{-i\omega t}| + 2|g_2(t)\rangle\langle g_2(t)| + \sum_{j=3}^6 |g_j(t)\rangle\langle g_j(t)|}{1-p}
\end{aligned} \tag{6}$$

and with  $p = \langle g_1(t)|g_1(t)\rangle$  denoting the apriori probability of the pure field state  $|g_1(t)\rangle$ . In general, the quantum

states  $\hat{\rho}_1$  and  $\hat{\rho}_2$  are not orthogonal so that they cannot be distinguished by any quantum measurement perfectly [12–14].

Therefore, in order to optimize the postselection of a perfectly entangled Bell state  $|\Psi^+\rangle_{AB}$  it is necessary to perform a POVM measurement on the optical radiation field with two possible measurement results  $m$ , say  $m \in \{1, 0\}$ . The first measurement result  $m = 1$  indicates an optimal projection of the field state  $\hat{\rho}_F(t)$  onto the pure field state  $\hat{\rho}_1$  and the second measurement result  $m = 0$ , indicates an optimal projection of  $\hat{\rho}_F(t)$  onto the mixed field state  $\hat{\rho}_2$ . Let us denote the positive operators associated with these two measurement results by  $\hat{T}_1 \geq 0$  and  $0 \leq \hat{T}_0 := \mathbf{I} - \hat{T}_1$ . ( $\mathbf{I}$  denotes the unit operator in the Hilbert space of the single-mode radiation field.) The positive operator  $\hat{T}_1$  of this POVM  $\{\hat{T}_1, \hat{T}_0\}$  has to be determined in such a way that for a given a priori probability  $p$  the error probability

$$E = p\text{Tr}\{\hat{\rho}_1\hat{T}_0\} + (1-p)\text{Tr}\{\hat{\rho}_2\hat{T}_1\} \quad (7)$$

is as small as possible. Diagonalizing the Hermitian operator  $\hat{A} := p\hat{\rho}_1 - (1-p)\hat{\rho}_2$  according to  $\hat{A} = \sum_{\lambda} \lambda |\lambda\rangle\langle\lambda|$  the solution of this optimization problem is given by the projection operator [9–11]

$$\hat{T}_1 = \sum_{\lambda \geq 0} |\lambda\rangle\langle\lambda| = \mathbf{I} - \hat{T}_0 \quad (8)$$

which projects onto the non-negative spectral components of the operator  $\hat{A}$ . The corresponding minimal error probability  $E_{min}$  of the optimal POVM measurement defined by Eq.(8) is determined by the trace norm distance between the two (unnormalized) components  $p\hat{\rho}_1$  and  $(1-p)\hat{\rho}_2$  of the quantum state  $\hat{\rho}_F(t)$ , i.e. [9, 11]

$$E_{min} = \frac{1}{2} (1 - \|p\hat{\rho}_1 - (1-p)\hat{\rho}_2\|_1). \quad (9)$$

The probability  $P_{Bell}$  with which this optimal POVM measurement of the optical radiation field prepares the distant material quantum systems  $A$  and  $B$  in the Bell state  $|\Psi^+\rangle$  it thus given by

$$P_{Bell} = p\text{Tr}_{field}\{\hat{\rho}_1\hat{T}_1\}. \quad (10)$$

From Eqs.(9) and (10) it is apparent that if the quantum states  $\hat{\rho}_1$  and  $\hat{\rho}_2$  were orthogonal the positive operator  $\hat{T}_1$  of the POVM measurement would be a projection operator onto the support of the state  $\hat{\rho}_1$  and the success probability  $P_{Bell}$  would be given by  $p$ . In addition, the

minimal error probability  $E_{min}$  would vanish. However, the typical non-orthogonality of the field states  $\hat{\rho}_1$  and  $\hat{\rho}_2$  complicates matters and causes unavoidable errors even if minimum-error POVM measurements are performed.

With the optimal POVM measurement  $\{\hat{T}_1, \hat{T}_0\}$  as defined by Eq.(8) we can associate a deterministic quantum operation [15] with the Kraus operators  $\{\hat{U}_1\sqrt{\hat{T}_1}, \hat{U}_0\sqrt{\mathbf{I} - \hat{T}_1}\}$  which allows to determine the quantum state of the matter-field system associated with the two measurement results  $m \in \{1, 0\}$  of the optimal POVM. Thereby, the linear operators  $\hat{U}_1$  and  $\hat{U}_0$  are partial isometries between the ranges of  $\sqrt{\hat{T}_1}$  and  $\sqrt{\hat{T}_0}$  and the Hilbert space of the optical radiation field. After a successful POVM measurement the state of both material quantum systems  $A$  and  $B$  is given by

$$\hat{\rho}_{AB}(t) = \frac{\text{Tr}_{field}\{|\Psi(t)\rangle\langle\Psi(t)|\hat{T}_1\}}{\text{Tr}_{A,B,field}\{|\Psi\rangle\langle\Psi|\hat{T}_1\}}. \quad (11)$$

Note that this quantum state of systems  $A$  and  $B$  is independent of the unitary transformation  $\hat{U}$ . Thus, the fidelity  $F_{opt}$  of an optimally prepared Bell pair which is postselected by a measurement result with value  $m = 1$  (corresponding to the POVM operator  $\hat{T}_1$ ) is given by

$$F_{opt} = \sqrt{\langle\Psi^+|\hat{\rho}_{AB}(t)|\Psi^+\rangle}. \quad (12)$$

In order to obtain some insight into the dynamical parameter regimes in which this postselection procedure may yield high-fidelity Bell pairs let us concentrate on the practically important case of large mean photon numbers, i.e.  $\bar{n} = |\alpha|^2 \gg 1$ , and on intermediate values of the interaction times  $\tau$  of the optical radiation field with the quantum systems  $A$  and  $B$  so that in Eq.(3) the photon-number dependent Rabi frequencies  $\Omega(n)$  can be linearized around the mean photon number  $\bar{n}$ . This linearization is valid if the condition

$$\frac{1}{2} \left| \frac{d^2\Omega}{dn^2}(n) \right|_{n=\bar{n}} \tau (\Delta n)^2 = \frac{1}{8\bar{n}} \frac{|\bar{\Omega}_0|^4 \tau}{|\bar{\Omega}|^3} \ll \pi \quad (13)$$

is fulfilled so that revival phenomena [8] can be neglected. Thereby,  $\Delta n = |\alpha| = \sqrt{\bar{n}}$  denotes the photon number uncertainty of the coherent state  $|\alpha\rangle$  and  $\bar{\Omega} = \sqrt{\Delta^2/4 + |g|^2\bar{n}}$  and  $\bar{\Omega}_0 = |g|\sqrt{\bar{n}}$  are the effective Rabi frequency and the resonant Rabi frequency associated with the mean photon number  $\bar{n}$ . In this linearization the field states of Eq.(3) can be approximated by

$$\begin{aligned}
|g_1\rangle &= |\alpha e^{-i\omega t} e^{i\theta}\rangle e^{i\omega_c \tau} \frac{1 + \Delta/(2\bar{\Omega})}{2\sqrt{2}} + |\alpha e^{-i\omega t} e^{-i\theta}\rangle e^{-i\omega_c \tau} \frac{1 - \Delta/(2\bar{\Omega})}{2\sqrt{2}}, \\
|g_2\rangle &= -e^{i\varphi} \frac{\bar{\Omega}_0}{4\bar{\Omega}} (|\alpha e^{-i\omega t} e^{i\theta}\rangle e^{i\omega_c \tau} - |\alpha e^{-i\omega t} e^{-i\theta}\rangle e^{-i\omega_c \tau}), \\
|g_3\rangle &= |\alpha e^{-i\omega t} e^{2i\theta}\rangle e^{2i\omega_c \tau} \frac{(1 + \Delta/(2\bar{\Omega}))^2}{8} + |\alpha e^{-i\omega t} e^{-2i\theta}\rangle e^{-2i\omega_c \tau} \frac{(1 - \Delta/(2\bar{\Omega}))^2}{8} + |\alpha e^{-i\omega t}\rangle \frac{1 - (\Delta/(2\bar{\Omega}))^2}{4}, \\
|g_4\rangle &= |g_5\rangle = -e^{i\varphi} \frac{\bar{\Omega}_0}{8\bar{\Omega}} \left( |\alpha e^{-i\omega t} e^{2i\theta}\rangle e^{2i\omega_c \tau} \left(1 + \frac{\Delta}{2\bar{\Omega}}\right) - |\alpha e^{-i\omega t} e^{-2i\theta}\rangle e^{-2i\omega_c \tau} \left(1 - \frac{\Delta}{2\bar{\Omega}}\right) - |\alpha e^{-i\omega t}\rangle \frac{\Delta}{\bar{\Omega}} \right), \\
|g_6\rangle &= e^{2i\varphi} \frac{\bar{\Omega}_0^2}{8\bar{\Omega}^2} (|\alpha e^{-i\omega t} e^{2i\theta}\rangle e^{2i\omega_c \tau} + |\alpha e^{-i\omega t} e^{-2i\theta}\rangle e^{-2i\omega_c \tau} - 2|\alpha e^{-i\omega t}\rangle)
\end{aligned} \tag{14}$$

with  $g = e^{i\varphi}|g|$ ,  $\theta = \bar{\Omega}_0^2 \tau / (2\bar{\Omega}\bar{n}) \ll 1$ , and with the modified effective Rabi frequency  $\omega_c = \bar{\Omega}[1 - \bar{\Omega}_0^2/(2\bar{\Omega}^2)]$ . This linearization implies that the apriori probability  $p$  entering Eq.(5) can be approximated by

$$p = \frac{\Delta^2}{8\bar{\Omega}^2} + \left( \frac{1}{4} - \left( \frac{\Delta}{4\bar{\Omega}} \right)^2 \right) \times (1 + \cos(2\bar{\Omega}\tau) \exp(-(\bar{\Omega}_0^2 \tau / (\bar{\Omega}\sqrt{\bar{n}}))^2 / 2)). \tag{15}$$

Furthermore, the overlaps between the coherent state  $|g_1(t)\rangle$  and the states  $|\alpha e^{i\omega t}\rangle$  and  $|g_3(t)\rangle$  constituting significant parts of the quantum state  $\hat{\rho}_2$  reduce to

$$\begin{aligned}
\langle \alpha e^{-i\omega t} | g_1(t) \rangle &= \frac{1}{\sqrt{2}} \exp(-(\bar{\Omega}_0^2 \tau / (\bar{\Omega}\sqrt{\bar{n}}))^2 / 2) \left( \cos(\bar{\Omega}\tau) + i \frac{\Delta}{2\bar{\Omega}} \sin(\bar{\Omega}\tau) \right), \\
\langle g_3(t) | g_1(t) \rangle &= e^{-[\bar{\Omega}_0^2 \tau / (2\bar{\Omega}\sqrt{\bar{n}})]^2 / 2} \left( e^{-i\bar{\Omega}\tau} \frac{(1 + \Delta/(2\bar{\Omega}))^3}{16\sqrt{2}} + e^{i\bar{\Omega}\tau} \frac{(1 - \Delta/(2\bar{\Omega}))^3}{16\sqrt{2}} \right) + \\
&\quad e^{-9[\bar{\Omega}_0^2 \tau / (2\bar{\Omega}\sqrt{\bar{n}})]^2 / 2} \left( e^{-3i\bar{\Omega}\tau} \frac{(1 + \Delta/(2\bar{\Omega}))^2 (1 - \Delta/(2\bar{\Omega}))}{16\sqrt{2}} + e^{3i\bar{\Omega}\tau} \frac{(1 + \Delta/(2\bar{\Omega}))(1 - \Delta/(2\bar{\Omega}))^2}{16\sqrt{2}} \right) + \\
&\quad e^{-[\bar{\Omega}_0^2 \tau / (2\bar{\Omega}\sqrt{\bar{n}})]^2 / 2} \left( e^{i\bar{\Omega}\tau} \frac{(1 + \Delta/(2\bar{\Omega}))[1 - (\Delta/(2\bar{\Omega}))^2]}{8\sqrt{2}} + e^{-i\bar{\Omega}\tau} \frac{(1 - \Delta/(2\bar{\Omega}))[1 - (\Delta/(2\bar{\Omega}))^2]}{8\sqrt{2}} \right) \tag{16}
\end{aligned}$$

From Eq.(16) it is apparent that in the case of resonant excitation, i.e.  $\Delta = 0$ , these overlaps vanish at interaction times  $\tau = (\pi + 2k\pi)/(2\bar{\Omega}_0)$  ( $k \in \mathbb{N}_0$ ). Furthermore, at these particular interaction times also all other overlaps  $\langle g_j(t) | g_1(t) \rangle$  with  $j = 2, 4, 5, 6$  vanish. Thus, in this linearization approximation the quantum states  $\hat{\rho}_1$  and  $\hat{\rho}_2$  are orthogonal at these interaction times so that they can be distinguished perfectly by a von Neuman measurement described by the projection operators  $\{\hat{T}_1 = \hat{\rho}_1, \hat{T}_0 = \mathbf{I} - \hat{T}_1\}$ . In this case the success probability reduces to  $p = (1 - \exp(-(\bar{\Omega}_0 \tau / \sqrt{\bar{n}})^2 / 2)) / 4$  and approaches the value of  $p = 1/4$  in the limit of sufficiently large interaction times of the order of the inverse vacuum Rabi frequency, i.e.  $\tau \gg \sqrt{\bar{n}}/\bar{\Omega}_0 = 1/\Omega_{vac}$ . In addition, the error probability  $E_{min}$  of this optimal von

Neuman measurement vanishes and the fidelity  $F_{opt}$  of the prepared Bell state  $|\Psi^+\rangle$  equals unity.

This resonant behavior at these particular interaction times is in marked contrast to the behavior at large detunings  $|\Delta/2| \gg |\bar{\Omega}_0|$  at which we obtain  $\bar{\Omega} \rightarrow |\Delta/2|$  so that the above mentioned overlaps tend to the non vanishing values  $|\langle \alpha e^{-i\omega t} | g_1 \rangle| = \exp\{-(\bar{\Omega}_0^2 \tau / (\bar{\Omega}\sqrt{\bar{n}}))^2 / 2\} / \sqrt{2}$  and  $|\langle g_3 | g_1 \rangle| = \exp\{-[\bar{\Omega}_0^2 \tau / (2\bar{\Omega}\sqrt{\bar{n}})]^2 / 2\} / (8\sqrt{2})$ . As a result, in this weak coupling limit it is only for extremely large interaction times, i.e.  $\tau \geq |\Delta/\bar{\Omega}_0|/\Omega_{vac} \gg 1/\Omega_{vac}$ , that these overlaps become small. However, at these interaction times, which are significantly larger than the inverse vacuum Rabi frequency, typically additional effects originating from spontaneous emission of photons also have

to be taken into account which have been neglected in our analysis. Thus, apart from these extremely large interaction times in the weak coupling limit it is never possible to distinguish the relevant field states  $\hat{\rho}_1$  and  $\hat{\rho}_2$  perfectly so that the preparation of perfect Bell states  $|\Psi^+\rangle$  is impossible.

Physically speaking, these marked differences between the strong and the weak coupling cases are due to the characteristic dephasing phenomena which are also responsible for the well known collapse phenomena in the Jaynes-Cummings-Paul model [8]. For the case of interaction times  $\tau$  characterized by the inequalities (13) these interference phenomena are captured quantitatively by the various slightly shifted coherent states entering Eq.(16). Although these coherent states of the form  $|\alpha e^{-i\omega t} e^{i\theta k}\rangle$  with  $\pm k = 0, 1, 2$  are shifted in their phases only slightly by multiples of the small amount  $\theta = \bar{\Omega}_0^2 \tau / (2\bar{\Omega}\sqrt{\bar{n}}) \ll 1$  their overlaps are given by

$$|\langle \alpha e^{-i\omega t} e^{-ik\theta} | \alpha e^{-i\omega t} e^{-ik'\theta} \rangle| = \exp\{ -[(k' - k)\bar{\Omega}_0^2 \tau / (\bar{\Omega}\sqrt{\bar{n}})]^2 / 2 \}. \quad (17)$$

This implies that for very small interaction times  $\tau$  or large mean photon numbers  $\bar{n}$ , i.e.  $\bar{\Omega}_0^2 \tau / (\bar{\Omega}\sqrt{\bar{n}}) / 2 \ll 1$ , the coherent states entering Eq.(16) are almost identical. Thus, in this limit the dynamics is well approximated by the semiclassical limit, i.e.  $\bar{n} \rightarrow \infty$ , in which the influence of the single-mode radiation field can be approximated well by a classical field and the influence of photon fluctuations is negligible. However, for intermediate interaction times of the order of  $\bar{\Omega}_0^2 \tau / (\bar{\Omega}\sqrt{\bar{n}}) / 2 \geq 1$  the overlaps of the coherent states entering Eq.(16) become small so that their interferences are suppressed significantly. This suppression of interference due to dephasing of these coherent states is also the reason for the appearance of the well known collapse phenomena in the Jaynes-Cummings-Paul model. In particular, in the case of resonant excitation it implies that even for arbitrary interaction times of the order of  $\tau \geq \sqrt{\bar{n}} / \bar{\Omega}_0 := 1 / \Omega_{vac}$  the field states  $\hat{\rho}_1$  and  $\hat{\rho}_2$  are almost orthogonal so that they can be distinguished almost perfectly by an appropriate quantum measurement. Effects of spontaneous emissions of photons from the excited state  $|2\rangle$  with a spontaneous decay rate  $\Gamma$  can still be neglected as long as the matter-field coupling as characterized by the vacuum Rabi frequency is sufficiently large so that  $\Omega_{vac} \gg \Gamma$ . Meeting these requirements is within reach of nowadays experimental possibilities. The recent experiment by Kimble et al. [16], for example, was performed in the optical frequency regime and was characterized by the parameters  $\Omega_{vac} / (2\pi) = 16\text{Mhz}$  and  $\Gamma / (2\pi) = 2,6\text{Mhz}$ . However, in cases of large detunings, i.e.  $|\Delta/2| \gg \bar{\Omega}_0$ , according to Eq.(16) strong dephasing requires significantly larger interaction times of the order of  $\tau \geq |\Delta/\bar{\Omega}_0| / \Omega_{cav} \gg 1 / \Omega_{vac}$  for which typically the influence of spontaneous decay processes can no longer be neglected.

In Figs. 2,3,4 numerical results are presented for characteristic quantitative measures which exhibit to which

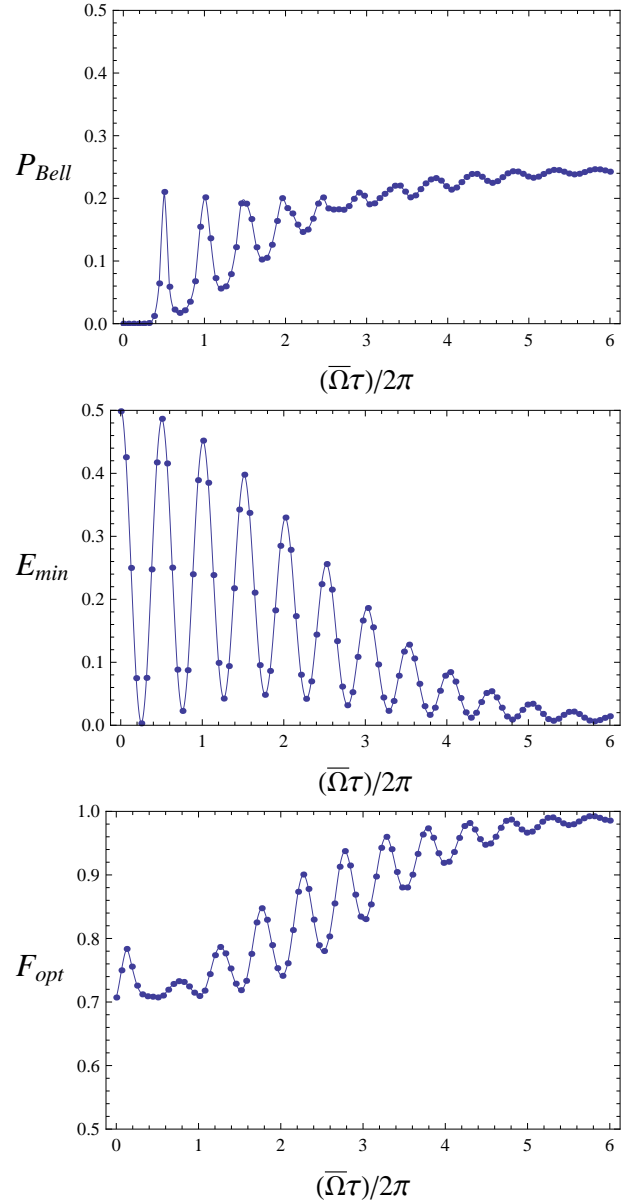


FIG. 2: Dependence of characteristic quantities on the interaction time  $\tau$  in the strong coupling limit  $\Delta = 0$ : The success probability for postselecting a Bell state  $|\Psi^+\rangle_{AB}$  of Eq.(10) (top); the minimum-error probability of the POVM measurement of the field of Eq.(9) (middle); the fidelity of the postselected Bell state  $|\Psi^+\rangle_{AB}$  of Eq.(12) (bottom).

extent postselection by a minimum-error POVM measurement on the optical radiation field is capable of preparing a Bell state  $|\Psi^+\rangle_{AB}$ . These numerical results are based on the exact quantum state of Eq.(3) from which the minimum-error POVM measurement is determined according to Eq.(8). This optimum minimum-error POVM depends on the characteristic electrodynamical interaction parameters involved, namely the interaction time  $\tau$ , the mean photon number  $\bar{n}$ , the detuning from resonance  $\Delta$ , and the strength of the quantum elec-

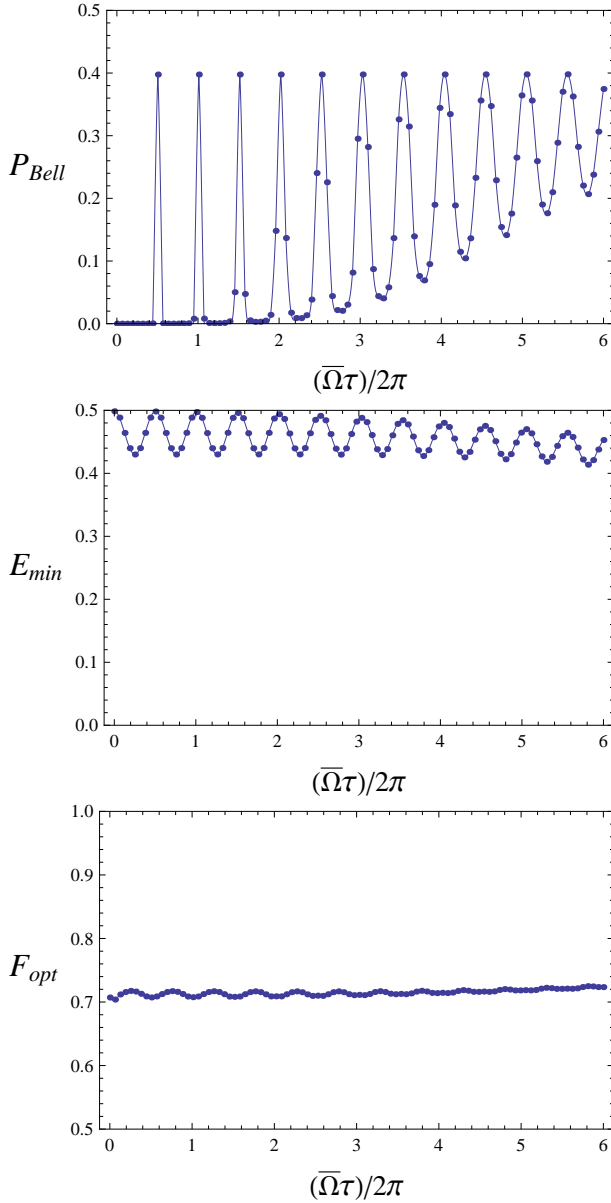


FIG. 3: Dependence of characteristic quantities on the interaction time  $\tau$  in the weak coupling limit  $\Delta = 5\bar{\Omega}_0$ : The success probability for postselecting a Bell state  $|\Psi^+\rangle_{AB}$  of Eq.(10) (top); the minimum-error probability of the POVM measurement of the field of Eq.(9) (middle); the fidelity of the postselected Bell state  $|\Psi^+\rangle_{AB}$  of Eq.(12) (bottom).

trodynamical coupling as measured by the resonant mean Rabi frequency  $\bar{\Omega}_0$ . In all these figures the mean photon number of the initially prepared coherent field state  $|\alpha\rangle$  is given by  $\bar{n} = |\alpha|^2 = 100$  so that typical quantum electrodynamical effects originating from photon number fluctuations as measured by  $\Delta n = \sqrt{\bar{n}} = 10$  are still apparent. In particular, this implies that deviations from the previously discussed analytical predictions of the linearization approximation are still observable.

Characteristic features of the strong coupling regime,

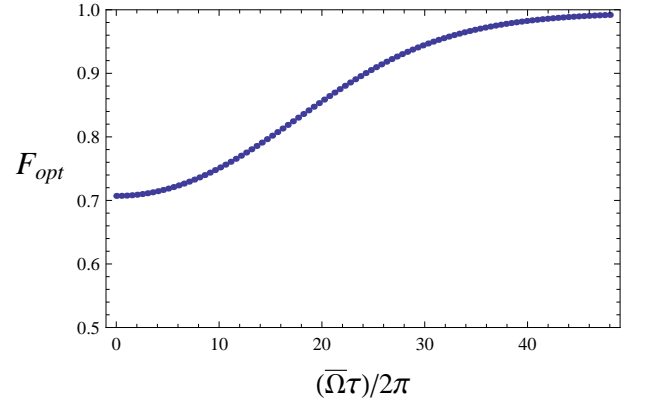


FIG. 4: Fidelity of the postselected Bell state  $|\Psi^+\rangle_{AB}$  of Eq.(12) in the weak coupling limit  $\Delta = 5\bar{\Omega}_0$  for long interaction times: The effects of dephasing lead to an asymptotic increase to unity

i.e.  $\Delta = 0$ , are depicted in Fig.2 as a function of the dimensionless parameter  $\bar{\Omega}\tau/(2\pi)$  which involves the interaction time  $\tau$  and the effective mean Rabi frequency  $\bar{\Omega}$ . Consistent with the approximate analytical expression for the a priori probability  $p$  as given by Eq.(15) for shorter interaction times  $\tau$  the success probability  $P_{Bell}$  of Eq.(10) exhibits maxima at integer multiples of the interaction time  $\tau = \pi/\bar{\Omega}$  (top). These maxima correspond to multiples of Rabi cycles at which the material three-level systems are found again in their initially prepared states  $|0\rangle$  and  $|1\rangle$  which constitute the qubits to be entangled. Correspondingly, also minima appear at odd integer multiples of the interaction time  $\tau = (\pi/2)/\bar{\Omega}$  at which the three-level systems involved populate the excited state  $|2\rangle$ . However, for larger interaction times these maxima and minima become less pronounced. This reflects the influence of the dephasing originating from the dependence of the Rabi frequencies  $\Omega(n)$  on the photon number  $n$  in Eq.(3) which is also responsible for the collapse phenomena in the Jaynes-Cummings-Paul model [8]. In the limit of large interaction times, i.e.  $\bar{\Omega}\tau \geq \sqrt{\bar{n}}$  the success probability approaches the value  $1/4$  consistent with Eq.(15). The minimum-error probability of Eq.(9) always exhibits maxima at completed Rabi cycles, i.e. at integer multiples of the interaction time  $\tau = \pi/\bar{\Omega}$  (middle). Thus, at these interaction times it is difficult to distinguish the field states  $\hat{\rho}_1$  and  $\hat{\rho}_2$  even by a minimum-error POVM measurement. This is due to the fact that in view of the periodic Rabi oscillations at these interaction times the atom-field state is similar to the initially prepared quantum state of Eq.(2) which is characterized by the property that the field states  $\hat{\rho}_1$  and  $\hat{\rho}_2$  are identical and thus indistinguishable. This is also the reason for the vanishing success probability at  $\tau = 0$ . However, due to dephasing the difference between minima and maxima of the minimum-error probability eventually vanishes in the limit of sufficiently large interaction times. Similarly, also the fidelity of a Bell pair



which is postselected by such a minimum-error POVM field measurement exhibits periodic oscillations with the Rabi frequency  $\bar{\Omega}$  (bottom). The maxima of these oscillations appear at odd integer multiples of the interaction time  $\tau = (\pi/2)/\bar{\Omega}$ . At these interaction times the three-level systems are likely to be found in the excited states  $|2\rangle$  so that we expect small success probabilities at these interaction times. However, due to dephasing and the related collapse phenomena at sufficiently long interaction times the differences between minima and maxima of the fidelity eventually tend to zero and the achievable fidelity approaches its maximum possible value of unity. These sufficiently long interaction times of the order of  $\tau \geq \sqrt{n}/\bar{\Omega}_0 = 1/\Omega_{vac}$  are therefore most favorable for preparing a high-fidelity material Bell pair in the Bell state  $|\Psi^+\rangle_{AB}$  provided spontaneous emission of photons into other modes of the radiation field is negligible. Recent quantum electrodynamical experiments performed in the strong coupling regime [16] demonstrate that such large vacuum Rabi frequencies are within nowadays experimental possibilities.

This strong coupling behavior is in marked contrast to the dependence of these characteristic quantities on the interaction time in the weak coupling limit in which the detuning is large, i.e.  $|\Delta| \gg \bar{\Omega}_0$ . This case is depicted in Figs.3 and 4. From the interaction times shown in Fig.3 characteristic oscillations of these quantities with the mean Rabi frequency  $\bar{\Omega} \rightarrow |\Delta/2|$  are apparent. They originate from the instantaneous turn on and turn off of the interactions between the optical radiation field and the quantum systems  $A$  and  $B$ . Although at their maxima the success probabilities  $P_{Bell}$  are slightly larger than in the resonant case of Fig.2 the fidelity of the postselected Bell pairs is significantly smaller and oscillates slightly around the value of  $F_{opt} = 1/\sqrt{2}$ . In view of the large detuning considered effects of dephasing are negligible for the interaction times depicted in Fig.3. Effects of dephasing become important only in the limit of extremely long interaction times of the order of  $\tau \geq |\Delta/\bar{\Omega}_0|/\Omega_{cav} \gg 1/\Omega_{vac}$ . The resulting collapse phenomena cause an increase of the achievable fidelities so that asymptotically they approach the maximum possible value of unity (compare with Fig.4). However, typically at these extremely long interaction times the influence of spontaneous emission of photons into other modes of the radiation field is no longer negligible and our theoretical model is no longer adequate for describing such cases. Therefore, for the preparation of high-fidelity Bell states by postselection the weak coupling regime exhibits clear limitations even if the postselection is performed by minimum-error POVM measurements.

### III. EFFECTS OF PHOTON LOSS

In this section we investigate the effects of photon loss during the propagation of the single-mode optical radiation field through the optical fiber. Modeling the result-

ing propagation-induced photon loss of this single-mode field by the dynamics of a damped harmonic oscillator characteristic features of entanglement creation by postselection are discussed. It is demonstrated that in the strong coupling regime of resonant interactions besides an overall decay of success probabilities and achievable fidelities of the postselected Bell pairs also characteristic interference oscillations appear which originate from dephasing and the associated collapse phenomena.

Let us consider again the quantum electrodynamical model of Sec.II A with the only difference that during the propagation of the optical radiation field through the fiber the dynamics is described by a damped harmonic oscillator. Thus, during the time interval  $(\tau, T + \tau)$  the free dynamics of the optical radiation field previously described by the Hamiltonian  $\hat{H}_F = \hbar\omega\hat{a}^\dagger\hat{a}$  is replaced by the Lindbladian master equation

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}_F, \hat{\rho}] + [\hat{L}, \hat{\rho}\hat{L}^\dagger] + [\hat{L}\hat{\rho}, \hat{L}^\dagger] = \mathcal{L}\hat{\rho} \quad (18)$$

for the field state  $\hat{\rho}(t)$  with the Lindblad operator  $\hat{L} = \sqrt{\gamma/2}\hat{a}$ . Thereby, the damping rate  $\gamma$  with its associated quality factor  $Q = \omega/\gamma$  characterizes the photon loss in the optical fiber.

A general solution of this equation can be obtained with the help of a Laplace transformation

$$\hat{\rho}(z) := \int_0^\infty \hat{\rho}(t)e^{-zt} dt \quad (19)$$

which transforms the master equation with the initial condition  $\hat{\rho}(t=0)$  into an algebraic equation for  $\hat{\rho}(z)$ . Inverting its solution with the help of the inverse relation

$$\hat{\rho}(t) = \frac{1}{2\pi i} \int_{\mathcal{C}} e^{zt} \hat{\rho}(z) dz \quad (20)$$

we obtain the corresponding solution  $\hat{\rho}(t)$  for the field state at time  $t$ . Thereby, the path of integration  $\mathcal{C}$  has to be chosen in such a way that all poles of  $\hat{\rho}(z)$  are included. Thus, in the number representation, i.e.  $\hat{\rho}(t) = \sum_{n,m=0}^\infty \rho_{n,m}(t) \exp[-i\omega(n-m)t] |n\rangle\langle m|$ , the time dependent solution of the master equation (18) is given by

$$\begin{aligned} \rho_{n,m}(t) &= e^{-\gamma(m+n)t/2} \sum_{i=0}^\infty \rho_{n+i,m+i}(t=0) \times \\ &\times \frac{\sqrt{(n+i)!} \sqrt{(m+i)!} (1-e^{-\gamma t})^i}{\sqrt{n!} \sqrt{m!} i!}. \end{aligned} \quad (21)$$

With the help of this solution it is straightforward to propagate any field coherence, say  $|n\rangle\langle m|$ , from time  $\tau$ , i.e. immediately after the interaction of the optical field with quantum system  $A$ , to time  $T + \tau$ , i.e. after completion of the propagation through the fiber. Thus, a coherence between coherent states, such as  $|\beta\rangle\langle\alpha|$ , for example, evolves to a coherence of the form

$$e^{\mathcal{L}T} |\beta\rangle\langle\alpha| = e^{-(1-e^{-\gamma T})(\frac{|\alpha|^2+|\beta|^2}{2}-\beta\alpha^*)} |\beta e^{-\gamma T/2}\rangle\langle\alpha e^{-\gamma T/2}|. \quad (22)$$

In Fig.5 numerical results are presented which demonstrate characteristic properties of the postselection of a Bell state  $|\Psi^+\rangle_{AB}$  by a minimum-error POVM measurement of the optical radiation field in the presence of photon loss during propagation through the optical fiber. Thereby, the quantum state resulting from all interactions between the optical field and the quantum systems  $A$  and  $B$  has been evaluated numerically with the help of relation (21). Apart from the photon loss during the propagation through the optical fiber and the choice of a fixed interaction time the parameters are the same as in Fig.2. The interaction time  $\tau = (23/4)2\pi/\Omega_0$  has been chosen in such a way that in the absence of photon loss the fidelity of creating a Bell state has a maximum and that its value is close to unity (compare with Fig.2). Fig.5 depicts the dependence of the characteristic quantities  $P_{Bell}$ ,  $E_{min}$ , and  $F_{opt}$  of this minimum-error postselection procedure on the time  $T$  of propagation through the fiber. It is apparent that photon loss tends to decrease the success probability  $P_{Bell}$  and the fidelity  $F_{opt}$  of the postselected Bell pair. At the same time is also increases the minimum error  $E_{min}$ . For an optical fiber with an intensity loss of  $D$  dB/m propagation durations  $T$  can be translated into lengths  $L$  of the optical fiber by the relation  $L = (\gamma T)20/(D\ln 10)$ . Thus, at a wave length of 1550nm with a photon loss of 0.2dB/km, for example, the maximum value of  $\gamma T = 0.3$  depicted in Fig.5 corresponds to a fiber length of  $L = 13029$ m. It is interesting to note that all the characteristic quantities depicted in Fig.5 exhibit also an oscillatory behavior. It can be traced back to the fact that according to Eq.(22) for small photon loss, i.e.  $\gamma T \ll 1$ , all relevant field coherences between coherent states also involve a characteristic frequency  $\tilde{\omega} = \gamma \text{Im}(\alpha^* \beta)$ . According to Eq.(14) due to dephasing typical relevant coherent states are of the form  $|\alpha e^{ik\theta}\rangle$  with  $k = \pm 1, \pm 2, \dots$ . Therefore, an estimate of these characteristic frequencies can be obtained by assuming that  $\alpha = \sqrt{n}e^{i\theta}$  and  $\beta = \sqrt{n}$ , for example, which yields an oscillation frequency of the order of  $\tilde{\omega} = \gamma n(\Omega_0 \tau / (2n)) = \Omega_0 \gamma \tau / 2$  for resonant coupling, i.e.  $\Delta = 0$ .

#### IV. SUMMARY AND CONCLUSION

In generalization of the hybrid quantum repeater model of van Loock et al. [1, 6, 7] we have studied the possibilities of preparing high-fidelity Bell pairs of two material qubits by postselection with the help of a single-mode optical radiation field. Whereas the original proposal of van Loock et al. [1] concentrated on the dynamical regime of weak coupling, in which the interaction between the optical radiation field and the two material quantum systems involved can be described perturbatively, our discussion concentrated on the strong coupling regime of almost resonant interaction. For this purpose we determined the optimum POVM measurements which have to be performed on the single-mode optical radia-

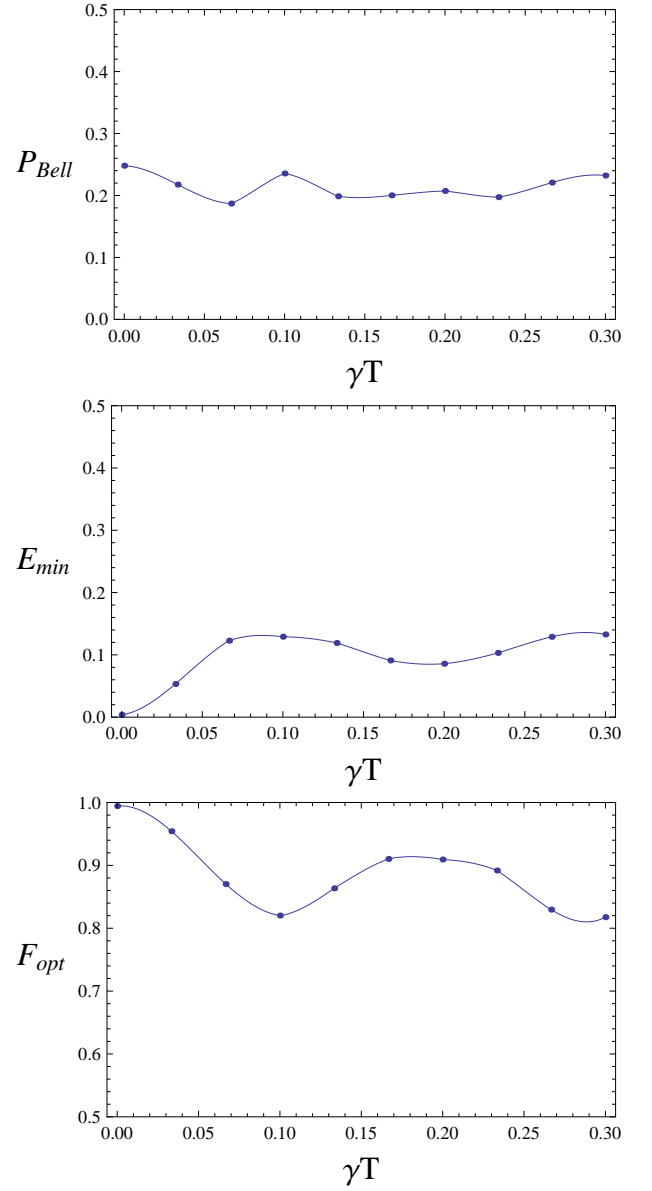


FIG. 5: Dependence of characteristic quantities on the propagation time  $T$  (in units of  $1/\gamma$ ) through a lossy optical fiber in the strong coupling limit  $\Delta = 0$  for an interaction time  $\tau = (23/4)2\pi/\Omega_0$ : The success probability for postselecting a Bell state  $|\Psi^+\rangle_{AB}$  of Eq.(10) (top); the minimum-error probability of the POVM measurement of the field of Eq.(9) (middle); the fidelity of the postselected Bell state  $|\Psi^+\rangle_{AB}$  of Eq.(12) (bottom).

tion field in order to postselect a Bell pair with minimum error. On the basis of this analysis we demonstrated that some of the limitations of the non-resonant weak coupling limit can be circumvented successfully in the strong coupling regime. In particular, provided the propagation of the optical radiation field through the fiber is ideal it is possible to create Bell pairs of fidelities arbitrarily close to unity provided the interaction time between the ma-

terial quantum systems and the optical field is chosen properly. This is due to the fact that at these particular interaction times the quantum states of the optical radiation field which are entangled with the desired two-qubit Bell state and with the other material quantum states involved are almost orthogonal so that they can be distinguished almost perfectly by a von Neuman measurement. This is in marked contrast to the weak coupling regime of non-resonant interaction where the relevant field states are always non orthogonal so that they cannot be distinguished perfectly by any quantum measurement and the resulting postselection is never perfect. Furthermore, due to photon-induced dephasing effects which are characteristic for the collapse phenomena of the Jaynes-Cummings-Paul model at these interaction times the success probabilities tend to the limiting value of  $1/4$  and are not significantly smaller than the corresponding values (of the order of 0.4) achievable in the weak coupling regime. We have also explored effects originating from photon loss taking place during the propagation of

the optical radiation field through the optical fiber. In addition to an overall decrease of achievable fidelities and success probabilities, in the strong coupling regime also an oscillatory behavior is apparent which originates from dephasing and the connected collapse phenomena. Thus, the strong coupling regime of the hybrid quantum repeater model of van Loock et al. [1] offers interesting perspectives for entangling material quantum systems over not too long distances and thus for providing high-fidelity Bell pairs at high repetition rates for future realizations of quantum repeaters.

## V. ACKNOWLEDGEMENT

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- [1] P. van Loock, T. D. Ladd, K. Sanaka, F. Yamaguchi, K. Nemoto, W. J. Munro, and Y. Yamamoto, Phys. Rev. Lett. **96**, 240501 (2006).
  - [2] Th. Beth and G. Leuchs eds., *Quantum Information Processing* (Wiley-VCH, Weinheim, 2005).
  - [3] N. Sangouard, C. Simon, H. de Riedmatten, and N. Gisin, Rev. Mod. Phys. **83**, 33 (2011).
  - [4] H.-J. Briegel, W. Dür, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **81**, 5932 (1998).
  - [5] W. Dür, H.-J. Briegel, J. I. Cirac, and P. Zoller, Phys. Rev. A **59**, 169 (1999).
  - [6] T. D. Ladd, P. van Loock, K. Nemoto, W. J. Munro, and Y. Yamamoto, New J. Phys. **8**, 184 (2006).
  - [7] P. van Loock, N. Lütkenhaus, W. J. Munro, and K. Nemoto, Phys. Rev. A **78**, 062319 (2008).
  - [8] W. P. Schleich, *Quantum Optics in Phase Space* (Wiley-VCH, Weinheim, 2001).
  - [9] C. W. Helstrom, *Quantum Detection and Estimation Theory* (Academic, New York, 1976).
  - [10] A. S. Holevo, Trans. Moscow Math. Soc. **26**, 133149 (1972).
  - [11] M. Hayashi, *Quantum Information* (Springer, Berlin, 2006).
  - [12] A. Chefles, Contemp. Phys. **41**, 401 (2000).
  - [13] A. Chefles, Phys. Lett. A **239**, 399 (1998).
  - [14] M. Kleinmann, H. Kampermann, and D. Bruss, Phys. Rev. A **81**, 020304 (2010).
  - [15] M.A. Nielsen and A. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge, Cambridge, 2000).
  - [16] J. McKeever, A. Boca, A. D. Boozer, J. R. Buck, and H. J. Kimble, Nature **425**, 268 (2003).